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# Single top quark $tW + X$ production at the LHC: a closer look

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## ABSTRACT

We have reexamined the  $tW + X$  single top-quark production process which is important at the LHC contrary to the Tevatron. The special attention was paid to the treatment of the  $2 \rightarrow 2[Wt]$  process and the part of it's next-to-leading correction:  $2 \rightarrow 3[Wtb]$  process. We show that  $2 \rightarrow 3[Wtb]$  process has to be correctly taken into account with a proper subtraction of the top pair contribution and that it has qualitatively different kinematical distributions from the  $2 \rightarrow 2[Wt]$  process. We present the total cross section of the  $tW + X$  production to be about 62 pb at QCD scale be taken as a top quark mass, suggest the method of combining  $Wt$  and  $Wtb$  processes with gauge invariant subtraction of the  $t\bar{t}$  part which allows to reproduce correct kinematical properties and perform a proper event simulation of the  $tW + X$  process in the whole kinematical region.

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# 1 Introduction

The study of the electroweak single top-quark physics is a very important part of research programs at future TeV energy colliders. Such a study allows to investigate with high enough accuracy properties of the top-quark and to measure a  $Wtb$  coupling structure. It may shed a light on the underlying theory which probably stands beyond the Standard Model [1, 2, 3, 5]. Besides, single top production at LHC has a large rate of the order of 300 pb and therefore it gives an important part of the background to various "new physics" processes.

In this paper we concentrated on the  $pp \rightarrow tW + X$  production process at the LHC. This process was the subject of the previous studies [4, 6, 7]. In the paper [6] we have calculated  $tW + X$  process among the others processes important at the Tevatron and LHC colliders. In order to separate  $tWb$  process of the single top-quark production from the  $t\bar{t}$  pair production we have introduced a cut on the invariant  $Wb$  mass window ('*approach I*'). In the paper [7] the cross section of  $tW$  process was calculated in a different approach where  $t\bar{t}$  contribution was subtracted from  $tWb$  process in the narrow width approximation ('*approach II*'). However, both approaches have different drawbacks and have some aspects which have been treated not quite correctly. Therefore it deserves closer look at the process.

In '*approach I*' results are formally  $|m_{WB} - m_{top}|$  cut dependent. In the paper [6] this cut was chosen too modest from the experimental point of view ( $|m_{WB} - m_{top}| < 3\Gamma_t$ ) since the real experimental mass window for the top-quark is of the order of 20 GeV  $\simeq 10 - 15\Gamma_t$ . This was pointed out in the paper [7]. However, in the present study we give the pure theoretical arguments how the cut should be chosen and explain why it has to be significantly larger than the top width. This cut should be of the order of 20 GeV even in case of an ideal detector. After that it will be clear that a formal cut dependence is significantly reduced. Specially one should stress that the ('*approach I*') reproduces *correctly* kinematical distributions which is the crucial point for a phenomenological analysis.

The '*approach II*' in it's turn is cut independent but it does not have receipt how to simulate  $Wtb$  events at all. In the paper [7] the only  $2 \rightarrow 2[Wt]$  process ( $bg \rightarrow Wt$ ) has been used for an event simulation. However in this case an important part of  $Wtb$  events are absent and such an implementation of the '*approach II*' leads to the wrong kinematical distributions for  $tW + X$  process. For the numerical values of the cross section the QCD scale  $\hat{s}$  has been used. But one should point out that such a scale is too large and leads to significantly lower rate even for top quark pair production as we know from NLO calculations [10]. For single top  $Wt$  process NLO results have not been obtained yet, however one would expect lower characteristic scale comparing to top pair.

In this paper we have been developed the '*approach I*'. We apply the reasonable  $|m_{WB} - m_{top}|$  cut consistent with theoretical arguments and an experimental mass resolution for top-quark. We have calculated also the cross sections using '*approach II*' for the cross check.

For both methods one needs to apply subtraction procedure to avoid double counting. We have suggested the new procedure of the combining of the two different processes. This method implies the correct subtraction procedure, reproduces the correct kinematical properties in the whole phase space region, and gives stable results.

Our paper is organised as follows. In the section II we compare two different approaches of a subtraction of  $t\bar{t}$  pair production from the process with  $tWb$  final state. In section III we develop the new approach for a treatment of double counting and combining two signal processes together. In section IV we present the final results and draw the conclusions.

## 2 Leading order $2 \rightarrow 2$ and $\mathcal{O}(1/\log m_t^2/m_b^2)$ $2 \rightarrow 3$ process: subtraction of $t\bar{t}$ pair production

In Fig. 1 we present the complete gauge invariant set of leading order and  $\mathcal{O}(1/\log m_t^2/m_b^2)$  diagrams contributing to the  $tW + X$  final state.

There are two problems one should avoid in order to combine correctly different contributions to the  $tW$  single top production: one should remove the contribution from  $t\bar{t}$  production, giving the same  $tWb$  final state and take care about the double counting which takes place when one simply adds contribution from two  $2 \rightarrow 2$  and  $2 \rightarrow 3$  processes.

This happens because  $2 \rightarrow 3$  process is singular in the region of collinear b-quarks coming from gluon splitting. The same singularity has been resolved for  $2 \rightarrow 2$  process when b-quark PDF was defined and collinear contributions of b-quark was resummed. The contribution from the collinear region should be taken only once, and therefore one should apply a subtraction procedure. It should be noticed that  $2 \rightarrow 2$  and  $2 \rightarrow 3$  processes have overlapping only for the leading log gluon splitting term.

In this section we would like to compare two different approaches of solving the first problem – subtracting the contribution from the top-quark pair production. As for double counting, we use the conventional solution in this section [6, 7], namely, we use the subtraction of gluon splitting term:

$$\sigma(gb + gg \rightarrow tW + X)_{real} = \sigma(gb \rightarrow tW) + \sigma(gg \rightarrow tW\bar{b}) - \sigma(g \rightarrow b\bar{b} \otimes gb \rightarrow tW) \quad (1)$$

As it was mentioned in the introduction, there are two basic approaches to remove the contribution from top-quark pair production in a gauge invariant way. The first ('*approachI*'), is the application of the cut on the invariant mass of  $Wb$ - pair in order to remove the resonant  $t\bar{t}$  contribution. This procedure is cut dependent, but it has the straightforward receipt how to simulate single-top quark production events with the proper kinematics. One should note however that the cut dependence is not arbitrary since the cut should be applied according to well known mass resolution [9] which is typically 10-15 GeV. In terms of the top-quark width the window cut should be applied to remove the  $t\bar{t}$  contribution would be of the order of  $\simeq \pm 20$  GeV  $\simeq 10 - 15\Gamma_{top}$ .

The second ('*approachII*') way of subtraction of  $t\bar{t}$  contribution is the narrow width limit approach [7]:

$$\sigma(gg \rightarrow tWb)_{singletop} = \sigma(gg \rightarrow tWb)_{total} - \sigma(gg \rightarrow t\bar{t}) * Br(t \rightarrow Wb) - interf[t\bar{t} \otimes tWb], \quad (2)$$

where  $interf[t\bar{t} \otimes tWb]$  means interference of  $t\bar{t}$  diagrams with the non-resonant ones. This procedure formally should reproduce the correct production rate for the single top quark. But from the practical point of view, it does not give any receipt how to simulate events of the single top-quark production which is crucial for the further kinematical studies.

Table 1 shows the results for two methods of subtraction of the  $t\bar{t}$  contribution. In order to give the idea how strong is the dependence on the  $Wb$  invariant mass we present numbers for two – 10 and  $15\Gamma_{top}$  window cuts which corresponds to  $\pm 16$  and 24 GeV mass windows respectively. All the numerical results have been obtained by means of the program CompHEP [8].

Results in the table are shown for several characteristic QCD factorisation/normalisation scales:  $Q = m_W, m_{top}, m_{top} + m_W$  GeV which give a natural scale interval for the process under study. We also show in the last column the results at  $Q^2 = \hat{s}$  for a comparison to previous calculations. From the table one can see that the subtraction term  $g \rightarrow b\bar{b} \otimes (gb + bg) \rightarrow tW^-$  is of the order of 80% of  $(gb + bg) \rightarrow W^-t$  cross section.

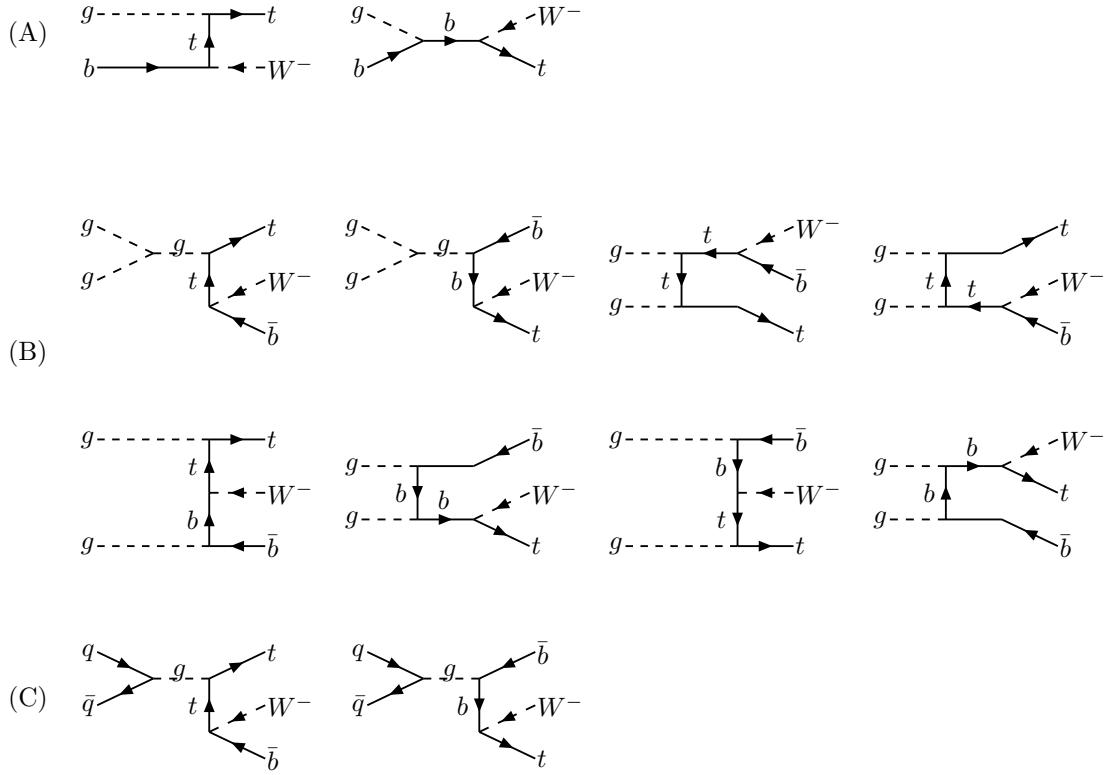


Figure 1: Diagrams for leading order  $2 \rightarrow 2$   $tW$  production (A) and  $\mathcal{O}(1/\log m_t^2/m_b^2)$   $2 \rightarrow 3$  process (B), (C)

One can also see that '*approach II*' of subtraction of  $t\bar{t}$  contribution gives cross section for  $W^-t + X$  process consistent with the '*approach I*' for  $\pm 15\Gamma_{top}$   $W^-b$  mass window cut. For example, one has 31.0 and 28.9 pb respectively for these two methods at  $\mu = m_{top}$ . Since the '*approach I*' is more physical in the sense that it allows to reproduce the correct kinematical distribution we use it with  $\pm 15\Gamma_{top}$   $W^-b$  mass window cut for the final results. One could also take a look at the  $Wb$  invariant mass distribution at the parton level which is presented in Fig. 2. From this figure one can clearly see that  $\simeq 25$  GeV  $Wb$  mass window would completely remove  $t\bar{t}$  contribution with its interference to the  $gg \rightarrow tW^-b$  processes.

One could easily perform the fitting procedure which leads to the more quantitative answer about the size of this window and gives those 25 GeV. This procedure significantly reduce the ambiguity of the choice of this window cut which is also of the order of the experimental mass resolution mentioned above.

Contribution from  $qq \rightarrow tWb$  process has been also taken into account in our study. The contribution from this process is not negligible and is of the order of 7% to the  $tWb$  final state after the removing  $t\bar{t}$  contribution.

One should also notice that cross sections are quite scale dependent. For three QCD scales:  $Q = m_W, m_{top}, m_{top} + m_W$  GeV the uncertainty due to the choice of different scales are of the order of 25-30%. The choice of the scale  $Q = \sqrt{s}$  gives significantly lower results. But we should stress that high QCD scale  $Q = \sqrt{s}$  seems to be unphysical since it gives almost factor two lower cross section even for  $t\bar{t}$  production at tree level in comparison with the next-to-leading(NLO) order result [10]. For  $Q = m_{top} = 175$  GeV tree level result is much close to

NLO one. Therefore it is quite reasonable to use  $Q = m_{top} = 175$  GeV choice for the processes involving single top-quark production for which the physical scale could be even smaller then for the  $t\bar{t}$  pair production.

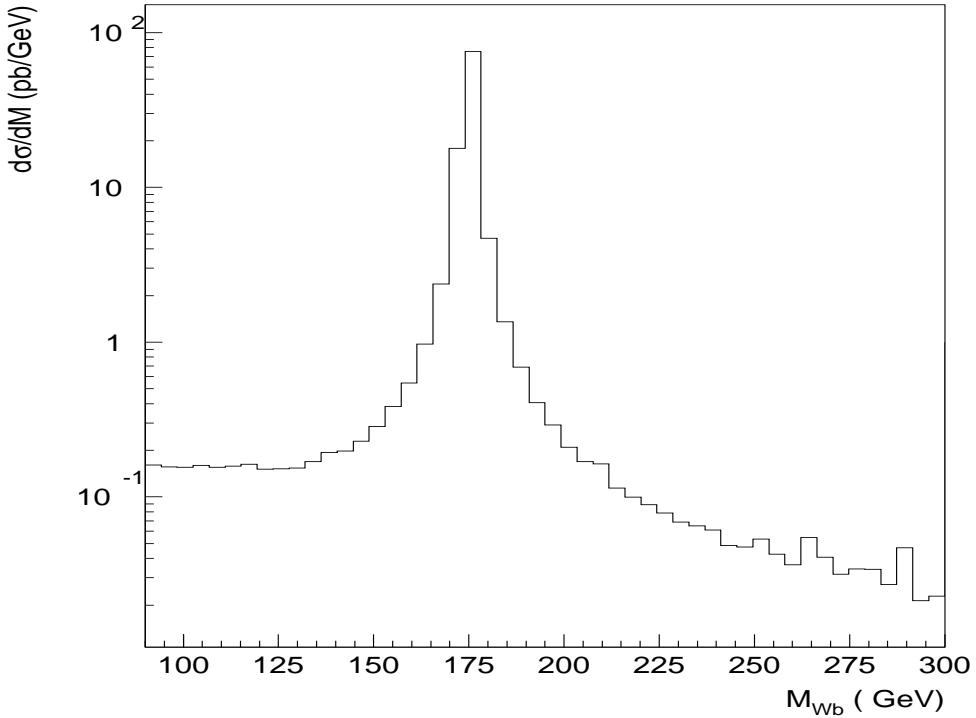


Figure 2: Parton  $Wb$  invariant mass distribution for  $gg \rightarrow tW^- \bar{b}$  processes.

### 3 Treatment of the double counting: comparison and combining of the $Wt+ISR$ and complete $tWb$ processes

In this section we would like make close look at the solution of the double counting problem.

Results from in Table 1 have been obtained using 'conventional' subtraction procedure. However one need to study how  $2 \rightarrow 2$  and  $2 \rightarrow 3$  processes should be combined in order to reproduce not only the total cross section but also the correct event kinematics.

We have used the following procedure to work out the receipt for this. We have compared various kinematical distributions of  $pp(bg) \rightarrow tW^- + b_{ISR} - 2 \rightarrow 2$  process with an additional b-quark from the initial state radiation and complete  $2 \rightarrow 3 - pp(gg + q\bar{q}) \rightarrow tW^- \bar{b}$  process. In this way one can try to find a proper matching between resummed contribution at the collinear region for the b-quark and complete tree level contribution at the hard region. Figure 3 shows transverse momenta and rapidity distributions for all three particles in the final state. As expected one can see the difference in the b-quark distributions. For  $pp(bg) \rightarrow tW^- + b_{ISR}$  process it is much softer and less central in comparison to the  $pp(gg + q\bar{q}) \rightarrow tW^- \bar{b}$  process. In the same time one can see that  $W - boson$  and  $t - quark$  distributions are nearly the same.

Since we know the absolute value of the combined cross section we propose the following method to match collinear and hard kinematical regions. One can use kinematical  $p_T^b$  separation of  $pp(bg) \rightarrow tW^- + b_{ISR}$  and

PROCESS	CS(pb), CTEQ4L			
	$\mu = m_W$	$\mu = m_{top}$	$\mu = m_{top} + m_W$	$\mu = \hat{s}$
$(gb + bg) \rightarrow W^- t$	29.4	29.1	28.6	27.8
$g \rightarrow b\bar{b} \otimes (gb + bg) \rightarrow tW^-$	25.3	23.9	22.9	21.8
$gg \rightarrow t\bar{t}$	717	523	457	358
interference $[t\bar{t} \otimes tWb]$	-14.6	-10.7	-9.19	-7.14
$gg \rightarrow tW^- b$ (no cuts)	737	536	469	368
$gg \rightarrow tW^- b$ ( $\pm 10\Gamma_{top}$ $W^- b$ mass cut)	42.7	30.9	27.0	20.8
$gg \rightarrow tW^- b$ ( $\pm 15\Gamma_{top}$ $W^- b$ mass cut)	33.3	24.3	21.2	16.5
$q\bar{q} \rightarrow t\bar{t}$	99.0	79.0	72.0	61.4
$q\bar{q} \rightarrow tW^- b$ (no cuts)	98.6	78.7	71.6	61.0
$q\bar{q} \rightarrow tW^- b$ ( $\pm 10\Gamma_{top}$ $W^- b$ mass cut)	2.9	2.3	2.1	1.7
$q\bar{q} \rightarrow tW^- b$ ( $\pm 15\Gamma_{top}$ $W^- b$ mass cut)	1.9	1.5	1.3	1.1
$(q\bar{q} + gg) \rightarrow t\bar{t}$	816	602	529	419
$(q\bar{q} + gg) \rightarrow tW^- b$ (no cuts)	836	615	541	429
$(q\bar{q} + gg) \rightarrow tW^- b$ ( $\pm 10\Gamma_{top}$ $W^- b$ mass cut)	45.6	33.2	29.1	22.5
$(q\bar{q} + gg) \rightarrow tW^- b$ ( $\pm 15\Gamma_{top}$ $W^- b$ mass cut)	35.2	25.8	22.5	17.6
$W^- t + [W^- tb]_{I(\pm 10\Gamma)} - [g \rightarrow b\bar{b} \otimes (gb + bg) \rightarrow tW^-]$	49.7	38.4	34.8	28.5
$W^- t + [W^- tb]_{I(\pm 15\Gamma)} - [g \rightarrow b\bar{b} \otimes (gb + bg) \rightarrow tW^-]$	<b>39.3</b>	<b>31.0</b>	<b>28.2</b>	<b>23.6</b>
$W^- t + [W^- tb]_{II} - [g \rightarrow b\bar{b} \otimes (gb + bg) \rightarrow tW^-]$	38.3	28.9	26.9	23.1

Table 1: cross section for various processes contributing to  $tW^- + X$  production with and without cut on the invariant  $Wb$  mass

$pp(gg + q\bar{q}) \rightarrow tW^- \bar{b}$  in the regions  $p_T^b < P_T^{cut}$  and  $p_T^b > P_T^{cut}$  respectively.

Now one can move the cut and try to satisfy two requirements, namely:

- 1) the common rate of  $pp(bg) \rightarrow tW^- + b_{ISR}$  with  $p_T^b < P_T^{cut}$  and  $pp(gg + q\bar{q}) \rightarrow tW^- \bar{b}$  with  $p_T^b > P_T^{cut}$  gives the combined total rate computed in previous section, in other words one can normalise a rate in a collinear region on the  $\sigma_{total} - \sigma[pp(gg + q\bar{q}) \rightarrow tW^- \bar{b}, p_T^b > P_T^{cut}]$ ;
- 2) the overall  $p_T^b$  distribution should be smooth.

The result is illustrated in Fig. 4 where we show several variants of combining those two processes for various values of  $P_T^{cut}$ . We have found that the optimal  $P_T^{cut}$  providing the smooth sewing for these two processes at the LHC is equal to 20 GeV. This value gives physically reasonable answer in which regions  $pp(bg) \rightarrow tW^- + b_{ISR}$  and  $pp(gg + q\bar{q}) \rightarrow tW^- \bar{b}$  processes should be considered.

We conclude that the method of combining of the  $p_T^b$  distribution of  $Wt+ISR$  gluon and complete tree level  $tWb$  process allows to find the physically motivated  $p_T$  cut on the  $b - quark$  which allows us to treat together those processes and simulate them in different kinematical regions of  $p_T^b$ .

We have estimated uncertainties due to a the choice of the QCD scale within the range  $M_W < \mu < M_{TOP} + M_W$  taking the central value of  $\mu = M_{TOP}$ . The total cross section presented in Table 1 is  $31.0^{+8.3}_{-1.8}$  pb within the QCD scale mentioned above.

## 4 Final results and conclusions

We have reexamined the  $tW + X$  single top-quark production process which is important at the LHC. We have shown that  $2 \rightarrow 3[Wtb]$  process has to be correctly taken into account with a proper subtraction of the top pair contribution and that it has qualitatively different kinematical distributions from the  $2 \rightarrow 2[Wt]$  process.

We suggest the new method of 'kinematical' sewing of two different processes contributing to the  $tW + X$  productions using the transverse b-quark momenta distribution. This method allows unambiguously simulate correct kinematical distribution of the total process of  $tW + X$  production in the whole kinematical region.

We have estimated the cross section of the single top  $tW + X$  production taking into account uncertainties

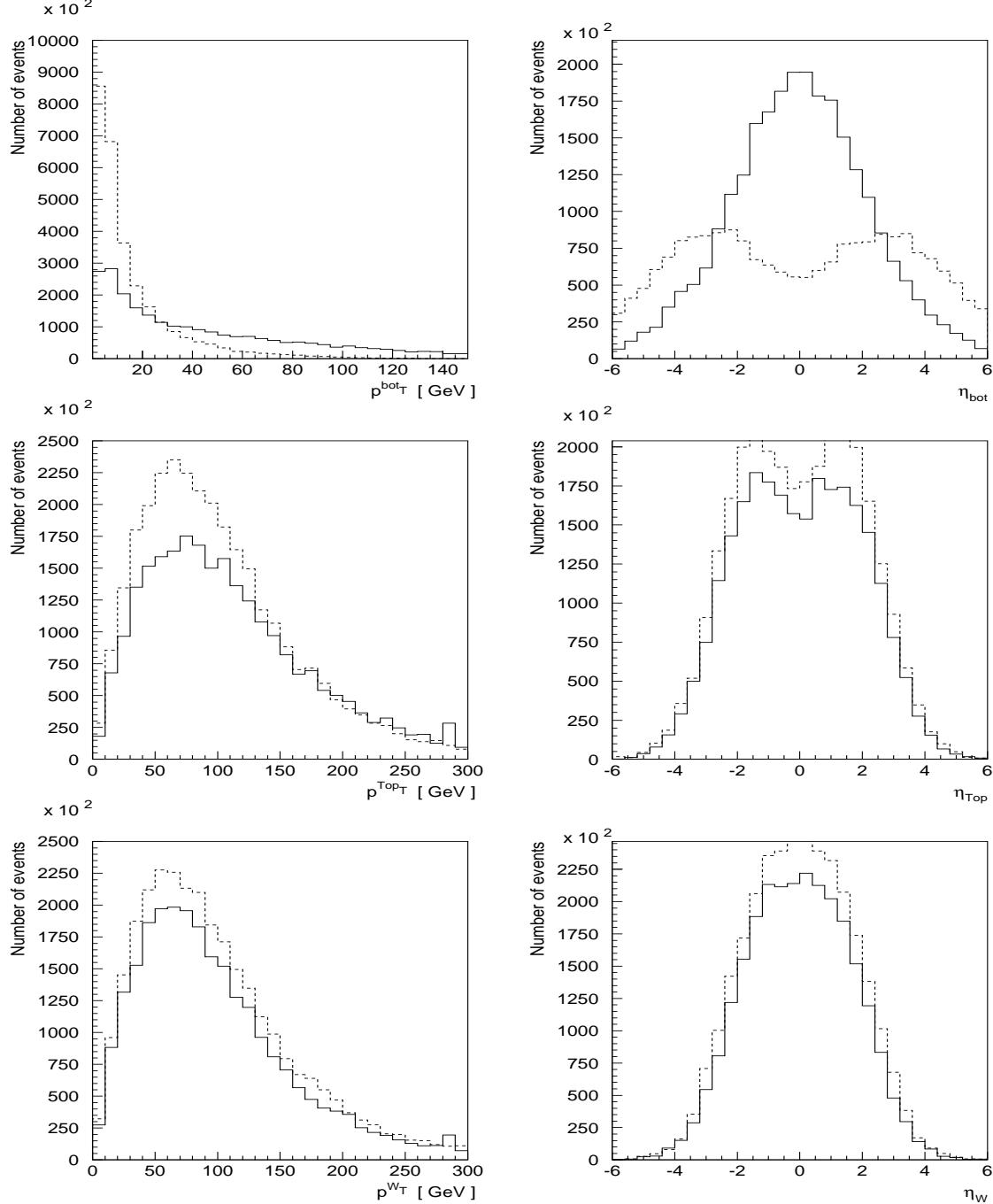


Figure 3: Transverse momenta and rapidity distributions for the final state particles of the processes  $pp(bg) \rightarrow tW -+ b_{ISR}$  (dashed line) – 2 → 2 process with additional b-quark from initial state radiation and true 2 → 3  $pp(gg + q\bar{q}) \rightarrow tW -\bar{b}$  process (solid line).

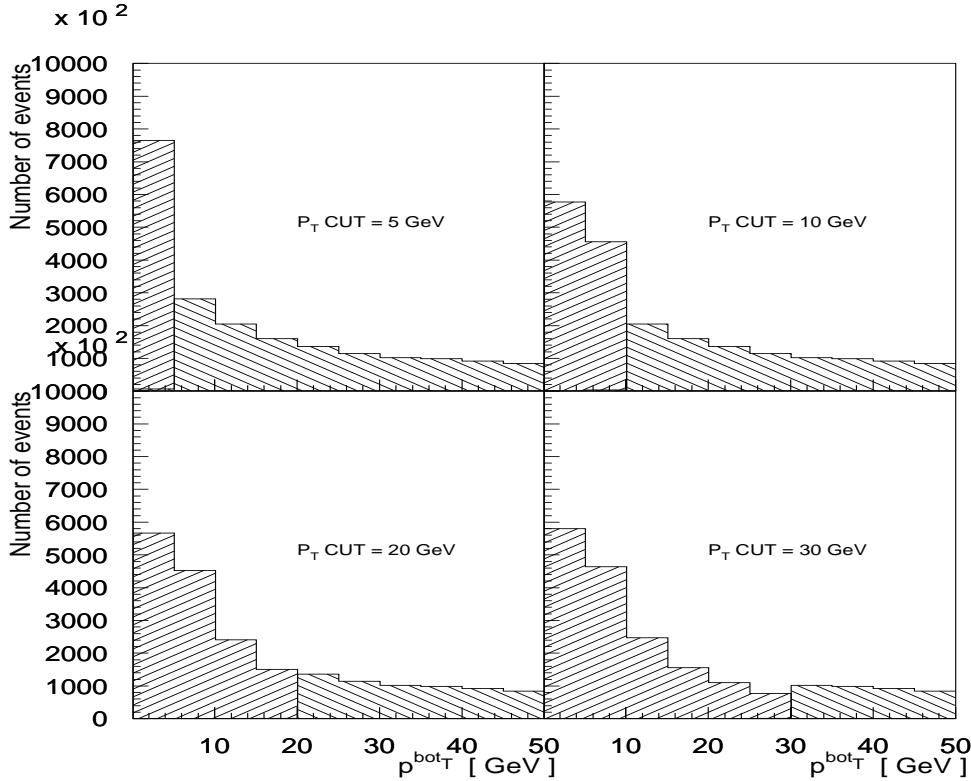


Figure 4: Transverse momenta distribution of the b-quark sewed for various values of  $P_T^{cut}$  for the  $pp(bg) \rightarrow tW^- + b_{ISR}$  and  $pp(gg + q\bar{q}) \rightarrow tW^- + \bar{b}$  processes.

due to the choice of the QCD scale. The cross section for single top and single anti-top quark production –  $tW^- + X$  and  $\bar{t}W^+ + X$  at the LHC are equal to each other in contrary to other processes of the single top-quark production. So, combined  $[tW^- + X + \bar{t}W^+ + X]$  cross section is  $62.0^{+16.6}_{-3.6}$  pb.

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